

EXERCISE SET 3: TRIGONOMETRY ANSWER KEY

No Calculator

1. **7/2 or 3.5** The discussion in Lesson 9 about the definition of the sine function and the unit circle made it clear that the value of the sine function ranges from -1 to 1 .

Therefore, the maximum value of $\frac{8 \sin 2x}{2} - \frac{1}{2}$ is $\frac{8(1)}{2} - \frac{1}{2} = \frac{7}{2}$ or 3.5 .

2. **1/36 or .027 or .028** An radian measure of $\pi/3$ is equivalent to 60° . If you haven't memorized the fact that $\cos(60^\circ) = 1/2$, you can derive it from the Reference Information at the beginning of every SAT Math section, which includes the 30° - 60° - 90° special right triangle. Since $a = 1/2$, $(a/3)^2 = (1/6)^2 = 1/36$.

3. **1.17**

$$(\sin x - \cos x)^2 = 0.83$$

FOIL:

$$\sin^2 x - 2\sin x \cos x + \cos^2 x = 0.83$$

Regroup:

$$\sin^2 x + \cos^2 x - 2\sin x \cos x = 0.83$$

Simplify:

$$1 - 2\sin x \cos x = 0.83$$

Subtract 1:

$$-2\sin x \cos x = -0.17$$

Multiply by -1 :

$$2\sin x \cos x = 0.17$$

Evaluate this expression:

$$(\sin x + \cos x)^2$$

FOIL:

$$\sin^2 x + 2\sin x \cos x + \cos^2 x$$

Regroup:

$$\sin^2 x + \cos^2 x + 2\sin x \cos x$$

Substitute:

$$1 + 0.17 = 1.17$$

4. **D** $\sin(\pi/6) = 1/2$ and $\cos(\pi/3) = 1/2$, so $\sin(\pi/6)/\cos(\pi/3) = 1$.

5. **D** If $\sin \theta < 0$, then θ must be either in quadrant III or in quadrant IV. (Remember that sine corresponds to the y -coordinates on the unit circle, so it is negative in those quadrants where the y -coordinates are negative.) If $\sin \theta \cos \theta < 0$, then $\cos \theta$ must be positive (because

a negative times a positive is a negative). Since $\cos \theta$ is only positive in quadrants I and IV (because cosine corresponds to the x -coordinates on the unit circle), θ must be in quadrant IV

6. **B** First, notice that a/b and b/a are reciprocals. Next, we can use the identity in Lesson 10 that

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

to see that choice (B) is just the reciprocal of $\sin x$. Alternately, we can just choose a value of x , like $x = 1$, and evaluate $\sin 1 = 0.841$. The correct answer is the expression that gives a value equal to the reciprocal of 0.841 , which is $1/0.841 = 1.19$. Plugging in $x = 1$ gives (A) 0.841 , (B) 1.19 , (C) 0.292 , (D) 0.540 .

7. **C** Recall from the Pythagorean Identity that $\cos b = \pm\sqrt{1 - \sin^2 b}$. Substituting $\sin b = a$ gives $\cos b = \pm\sqrt{1 - a^2}$. The angle $b + \pi$ is the reflection of angle b through the origin, so $\cos(b + \pi)$ is the opposite of $\cos b$, which means that $\cos(b + \pi) = \pm\sqrt{1 - a^2}$.

8. **D** Recall from the Pythagorean Identity that $\cos^2 x = 1 - \sin^2 x$.

$$\frac{\cos x}{1 - \sin^2 x} = \frac{3}{2}$$

Substitute $\cos^2 x = 1 - \sin^2 x$:

$$\frac{\cos x}{\cos^2 x} = \frac{3}{2}$$

Cancel common factor:

$$\frac{1}{\cos x} = \frac{3}{2}$$

Reciprocate:

$$\cos x = \frac{2}{3}$$