EXERCISE SET 1 ANSWER KEY

No Calculator

1a. **26** If the average of 4 numbers is 7.5, they must have a sum of $4 \times 7.5 = 30$. To maximize the range, we must maximize one of the numbers by minimizing the other 3 by setting them all equal to 1 (the smallest positive integer). The numbers therefore are 1, 1, 1, and 27, and the range is $27 - 1 = 26$.

1b. **1** To minimize the range, we "cluster" the numbers as closely together as possible. The tightest cluster of integers with a sum of 30 is 7, 7, 8, and 8, which gives a range of $8 - 7 = 1$.

2. **4.1** If the set contains four numbers, its median is the average of the middle two numbers, so the middle two numbers must have a sum of $(2)(4.2) = 8.4$. Thus the four numbers must be 2, 4, 4.4, and 6. (Notice that the question did not say that all numbers were integers.) The average of these is $16.4/4 = 4.1$.

3. **3.5** If the average of these numbers is 0, their sum must be $(4)(0) = 0$, and therefore $k = -15$ and the numbers, in increasing order, are −15, 2, 5, and 8. The median is $(2 + 5)/2 = 3.5$.

4. **8** $6 = 48/n$, so $n = 8$.

5. **8** $\frac{4+x}{2} = \frac{2+8+x}{3}$

Cross-multiply: $12 + 3x = 20 + 2x$

Subtract $2x$ and 12: $x = 8$

6. **46** The median divides the set into two equal parts, so 11 of these numbers must be less than 25 and 11 must be greater than 25. Since they are consecutive even integers, the 11 numbers above the median must be 26, 28, 30, 32, . . . 46.

7. **D** If p and q vary inversely, their product is a constant. $4 \times 6 = 24$, and the only other pair with a product equal to 24 is (D) 12 and 2.

8. **A** $n = sum/average = 12m/3k = 4m/k$

9. **A** The equation relating x and y is $y = k/x^2$. If $x = 1$, then $y = k$. If x is multiplied by 4, then $x = 4$ and $y = k/16$, so y has been divided by 16.

10. **C** $f(a, b) = Aa^2b^3 = 10$. $f(2a, 2b) = A(2a)^2(2b)^3 = 32(Aa^2b^3) = 32(10) = 320$.

11. **B** If this set has a mode of 7, then at least two of the numbers are 7. If the median is 4, then the two middle numbers must have a sum of $(2)(4) = 8$. Therefore the two middle numbers are 1 and 7, and the sequence must be n, 1, 7, 7. To maximize the average, we must maximize n, but n can't be 1, because then the set would not have a mode of 7. It must be the next lower integer, 0, and the average is $(0 + 1 + 7 + 7)/4 = 3.75$.

Calculator

12. **11** The only four numbers that satisfy these conditions are 1, 2, 4, and 4.

13. **24** If the average of five numbers is 10, their sum is $5 \times 10 = 50$. To maximize one, we must minimize the sum of the other four. If none is less than five, and all are different integers, they are 5, 6, 7, 8, and 24.

14. **5** If the variables vary inversely, their product is constant. $(0.5)(32) = 16$. The only pairs of positive integers with a product of 16 are (1, 16), (2, 8), (4, 4), (8, 2), and (16, 1).

15. **23** If the middle number is 28, there are five numbers less than 28, and five greater. Since they are consecutive integers, the least is $28 - 5 = 23$.

16. **2.5** Since $108 = A(3)^3$, $A = 4$, so if $62.5 = 4x^3$, $x = 2.5$.

17. **7** At least two of the integers must be 2 and none can be less than 1. If the sum must be $4 \times 3 = 12$, the set including the largest possible number is 1, 2, 2, and 7.

18. **7.5** The product of x and y is $2 \times 15 = 30$, so $y = 30/4 = 7.5$.

19. **C** Average $= (1 \times 4 + 2 \times 5 + 3 \times 4 + 4 \times 6 + 5 \times 5 + 6 \times 6)/30 = 3.7$. Median = average of 15th and 16th roll: $(4 + 4)/2 = 4$. $4 - 3.7 = 0.3$.

20. **A** y and x^2 must have a constant product of $4 \times 2^2 = 16$. Therefore, $y = 16/9$.

21. **B** Pick values for the original pressure and volume, such as 2 and 3. If they vary inversely, their product is the constant $2 \times 3 = 6$. If the pressure is increased by 50%, it becomes $(1.5)(2) = 3$, and so the volume becomes $6/3 = 2$, a change of $-33\ 1/3\%$.

22. **C** For both ordered pairs, $\frac{y}{\sqrt{x}}$ is a constant: $\frac{3}{\sqrt{4}} = \frac{6}{\sqrt{16}} = \frac{3}{2}$, so y is directly proportional to the square root of x.