EXERCISE SET 1 ANSWER KEY

Calculator

1. **6** $f(2) = 2^2 + 2 + k = 10$, so $6 + k = 10$ and $k = 4$. Therefore, $f(-2) = (-2)^2 + (-2) + 4 = 6$.

2. **4** The graph of the function $g(x) = 6 - h(x + 2)$ is the graph of h after (1) a shift 2 units to the left, (2) a reflection over the x-axis, and (3) a shift 6 units up. If we perform these transformations on the point $(-3, 2)$, we get the point $(-5, 4)$, and so the maximum value of g is 4 when $x = -5$.

3. **10**

$$g(3) = a(3) + b = 6$$
$$g(1) = a(1) + b = 7$$

Subtract the equations: $2a = -1$
Divide by 2: $a = -0.5$
Substitute to find b: $-0.5 + b = 7$
Add 0.5: $b = 7.5$
Therefore $g(x) = -0.5x + 7.5$

$$g(-5) = -0.5(-5) + 7.5 = 10$$

4. **48** $h(2) = f(g(2)) = f(2 + 5) = f(7) = (7)^2 - 1 = 48$

5. **5** $f(3) = 5$

6. **6** $f(k(6)) = f(4) = 6$

7. **2** $k(k(6)) = k(4) = 2$

8. **5** According to the table, the only input into k that yields an output of 5 is 1. Therefore, $f(x)$ must be 1, and the only input into f that yields an output of 1 is $x = 5$.

9. **A** Examination of the table reveals that, for all given values of x, $f(g(x)) = x$ and $g(f(x)) = x$. (This means that f and k are **inverse functions**, that is, they "undo" each other.) This implies that $f(k(x)) - k(f(x)) = x - x = 0$.

10. **D** One way to approach this question is to pick a new variable, z, such that $z = x - 1$ and therefore $x = z + 1$.

Original equation: $g(x - 1) = x^2 + 1$
Substitute $z = x - 1$: $g(z) = (z + 1)^2 + 1$
FOIL: $g(z) = z^2 + 2z + 1 + 1$
Simplify: $g(z) = z^2 + 2z + 2$
Therefore $g(x) = x^2 + 2x + 2$

11. **D**

$$f(h(x)) = f\left(\frac{x+1}{2}\right) = \left(\frac{x+1}{2} - 1\right)^2$$
$$= \left(\frac{x+1}{2} - \frac{2}{2}\right)^2 = \left(\frac{x-1}{2}\right)^2$$
$$= \frac{x^2 - 2x + 1}{4}$$

No Calculator

12. **2** The graph of g contains the point $(-1, 2)$, therefore $g(-1) = 2$.

13. **3** The graph of f contains the point $(3, 1)$; therefore, $f(3) = 1$, and so $g(f(3)) = g(1)$. Since the graph of g contains the point $(1, 3)$, $g(1) = 3$.

14. **2** The graph of g contains the point $(3, -1)$; therefore, $g(3) = -1$, and so $f(g(3)) = f(-1)$. Since the graph of f contains the point $(-1, 2)$, $f(-1) = 2$.

15. **8** The only input to function g that yields an output of -1 is 3. Therefore, if $g(f(x)) = -1$, $f(x)$ must equal 3. The only input to f that yields an output of 3 is -2, therefore $x = -2$ and $x + 10 = 8$.

16. **3** The only input for which f and g give outputs that are opposites is 3, because $f(3) = 1$ and $g(x) = -1$.

17. **1** The two points at which the graphs of g and f cross are $(-1, 2)$ and $(2, 1)$. Therefore, $a = -1$ and $b = 2$ and so $a + b = 1$.

18. **4** $h(x) = f(x) \times g(x)$ has a maximum value when $x = -1$, where $f(1) \times g(1) = 2 \times 2 = 4$.

19. **A** To graph $y = f(x) + g(x)$, we must simply "plot points" by choosing values of x and finding the corresponding y-values. For instance, if $x = -3$, $y = f(3) + g(3) = 4 + 0 = 4$, so the new graph must contain the point $(-3, 4)$. Continuing in this manner for $x = -2$, $x = -1$, and so on yields the graph in (A).