

Geometry SOL Review

Study Guide of Important Information

G.1 Logic

Conditional	Converse	Inverse	Contrapositive	Biconditional
$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$p \leftrightarrow q$

- Contrapositive is true when the conditional is true.
- Converse and inverse have the same truth value
- Additional symbols: "and" "or"

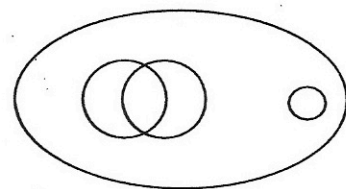
Law of Detachment

- *one conditional statement
- *second statement sounds like first statement hypothesis
- *conclusion sounds like first conclusion

Law of Syllogism

- *two conditional statements
- *first conclusion repeats as second hypothesis
- *conclusion is: If (1st hypothesis) then (2nd conclusion)

Venn Diagram



- *all small in large
- *some large in small
- *some of each overlap in the other
- *none when no overlap

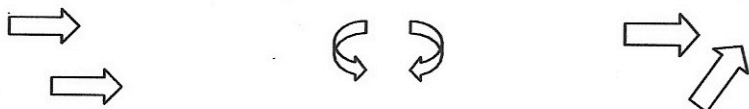
G.2 Coordinate Formulas and Transformations

Formulas:

Midpoint	Distance	Slope
$(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$	$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$	$\frac{y_2-y_1}{x_2-x_1}$

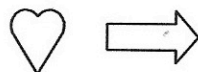
Transformations:

Translation (slide) Reflection (flip/fold) Rotation (spin/turn)

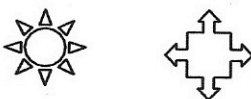


Symmetry:

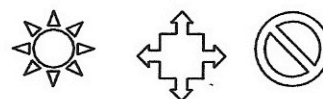
Line
– fold line; folds figure exactly in half, one half onto the other



Rotational
– spin figure by a degree value and figure matches onto itself



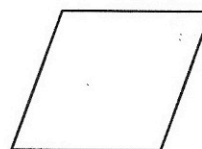
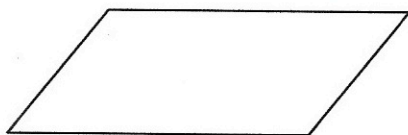
Point
– has rotational symmetry of 180°



Caution!

Parallelogram – point symmetry only!!!

Rhombus – line symmetry & point symmetry!!!



- *slopes of parallel lines are equal
- *slopes of perpendicular lines are negative reciprocals; product is -1

Vertical Lines:

Slope is undefined

Equation is $x = \#$

Horizontal Lines:

Slope is 0

Equation is $y = \#$

G.3 – Angle Relationships

Congruent Angles

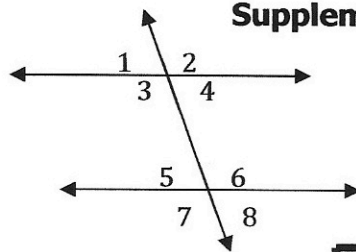
*(If lines are parallel)

Vertical $\angle 2 \cong \angle 3$

Alternate Interior $\angle 4 \cong \angle 5$

Alternate Exterior $\angle 1 \cong \angle 8$

Corresponding $\angle 3 \cong \angle 7$



Supplementary – sum of two angles is 180

Linear pair

$$m\angle 5 + m\angle 7 = 180^\circ$$

Consecutive Interior

$$m\angle 3 + m\angle 5 = 180^\circ$$

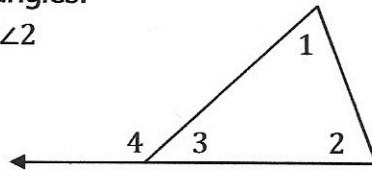
Triangles

Sum of interior angles is 180

$$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

Measure of an exterior angle is equal to the sum of its two remote interior angles.

$$m\angle 4 = m\angle 1 + m\angle 2$$



G.4 Ways to prove lines are parallel

- Alternate interior angles are congruent
- Corresponding angles are congruent
- Consecutive interior angles are supplementary
- The two lines are perpendicular to the same line

G.6 Triangle Inequalities

To form a triangle, sum of smaller two lengths must be greater than the largest

$$L > S + M \quad \text{triangle}$$

$$L = S + M \quad \text{flat}$$

$$L < S + M \quad \text{gap}$$

Largest angle is opposite largest side, smallest angle is opposite smallest side

Base angles of an isosceles triangle are congruent

Sides opposite congruent angles are congruent

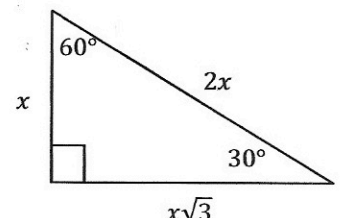
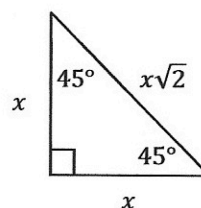
To find the possible lengths for the third side given the other sides:

subtract given #'s $< x <$ add given #'s

Special Right Triangles: only when given angle

45 – 45 – 90

30 – 60 – 90



G.5 Congruent and Similar Triangles

Congruent Triangles

- Corresponding angles are congruent
- Corresponding sides are congruent
- Ways to prove triangles are congruent
 - SSS, SAS, ASA, AAS, HL (for right triangles)

Similar Triangles

- Corresponding angles are congruent
- Corresponding sides are proportional
- Ways to prove triangles are similar
 - AA~, SSS~, SAS~

G.7 Right Triangles

Pythagorean Theorem $c^2 = a^2 + b^2$

*Used when two sides of a right triangle are given

Converse of the Pythagorean Theorem

*Identify a triangle as right, obtuse, or acute

$$c^2 = a^2 + b^2 \quad \text{Right}$$

$$c^2 > a^2 + b^2 \quad \text{Obtuse}$$

$$c^2 < a^2 + b^2 \quad \text{Acute}$$

*Check to see if triangle is possible

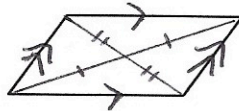
Right Triangle Trigonometry – SOH CAH TOA

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

G.8 Quadrilaterals

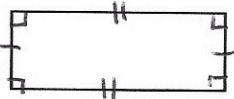
Parallelogram

Opposite sides are parallel
 Opposite sides are congruent
 Opposite angles are congruent
 Consecutive angles are supplementary
 Diagonals bisect each other



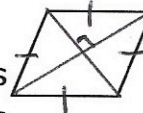
Rectangle

Parallelogram with:
 All right angles
 Diagonals are congruent



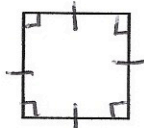
Rhombus

Parallelogram with:
 Four congruent sides
 Four congruent sides
 Diagonals are perpendicular
 Diagonals bisect opposite angles



Square

Parallelogram
 Rectangle
 Rhombus
 *all 10 properties listed above



Kite

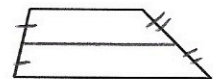
2 pair of adjacent sides are congruent
 No opposite sides are congruent



Trapezoid

Exactly one pair of opp. sides parallel
 Median joins midpoints of legs and is parallel to bases

$$m = \frac{1}{2}(b_1 + b_2)$$



Isosceles Trapezoid

Legs are congruent
 Pairs of base angles are Congruent
 Diagonals are congruent



G.9 Polygons

Formulas:

$(n - 2)180$	Sum of interior angles
$\frac{(n-2)180}{n}$	Each interior angle (regular)
360	Sum of exterior angles
$\frac{360}{n}$	Each exterior angle (regular)

*Exterior angle + interior angles = 180

*Exterior angle and its interior angle are supplementary

Tessellation Information

*Each vertex must have a sum of 360 degrees

Regular polygons that tessellate:

- Triangle – each angle measures 60°
- Square – each angle measures 90°
- Hexagon – each angle measures 120°

Combinations of regular polygons that tessellate

- square and octagon
- square and triangle
- triangle and hexagon

Other common regular polygon measurements (do not tessellate)

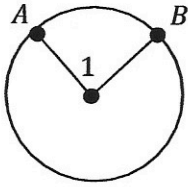
- Pentagon – each angle measures 108°
- Octagon – each angle measures 135°

**Non-regular figures can tessellate. Make sure that the sum of the angles at any vertex add to 360

G.10 Circles

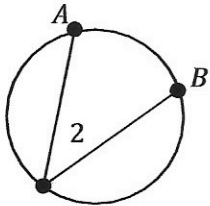
Angles & Arcs

Central Angle



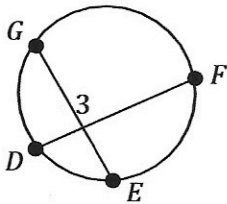
$$m\angle 1 = m\widehat{AB}$$

Inscribed Angle



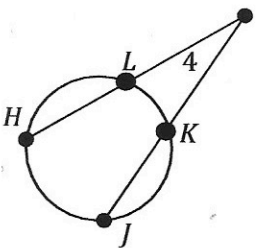
$$m\angle 2 = \frac{1}{2} m\widehat{AB}$$

Vertex inside circle



$$m\angle 3 = \frac{1}{2} (m\widehat{DE} + m\widehat{FG})$$

Vertex outside circle

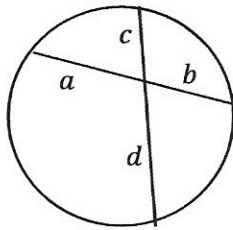


$$m\angle 4 = \frac{1}{2} (m\widehat{HJ} - m\widehat{LK})$$

Segments

Two Chords

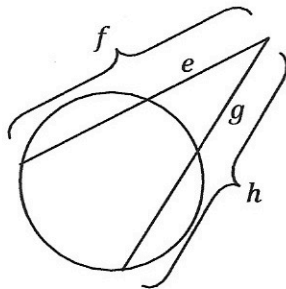
(product of segments from one chord = product of segments from the other)



$$ab = cd$$

Two Secants

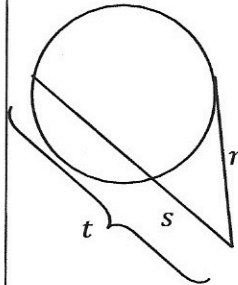
(outer secant segment₁ x whole secant₁ = outer secant segment₂ x whole secant₂)



$$ef = gh$$

Tangent and Secant

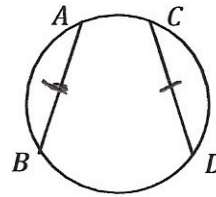
Tangent² = outer secant segment x whole secant



$$r^2 = st$$

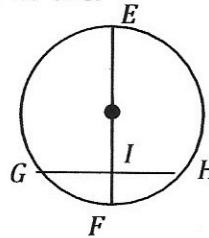
Miscellaneous Topics

Congruent chords have congruent arcs



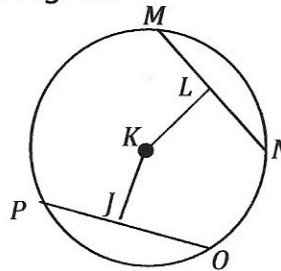
$$\overline{AB} \cong \overline{CD} \leftrightarrow \text{arc}AB \cong \text{arc}CD$$

A diameter perpendicular to a chord bisects the chord and its arc.



$$\begin{aligned} \overline{EF} \perp \overline{GH} &\leftrightarrow \overline{GI} \cong \overline{IH} \\ &\leftrightarrow \text{arc}GF \cong \text{arc}HF \\ &\leftrightarrow \text{arc}EG \cong \text{arc}EH \end{aligned}$$

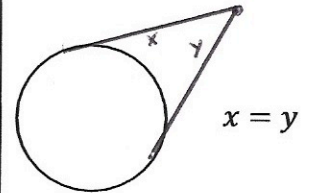
Chords equidistant from the center are congruent.



$$\begin{aligned} KL = JI &\leftrightarrow \overline{MN} \cong \overline{PO} \\ &\leftrightarrow \text{arc}MN \cong \text{arc}PO \end{aligned}$$

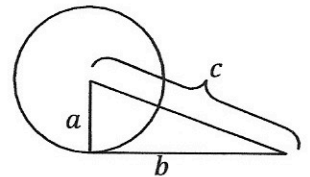
Miscellaneous Topics

Tangents from the same exterior point are congruent.



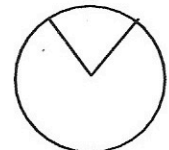
$$x = y$$

Tangent is perpendicular to the radius drawn to the point of tangency.



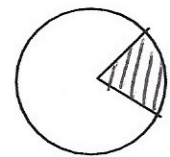
$$c^2 = a^2 + b^2$$

Arc Length



$$\frac{\text{degree}}{360} \cdot 2\pi r$$

Area of a sector



$$\frac{\text{degree}}{360} \cdot \pi r^2$$

G.13 Lateral Area, Surface Area & Volume of 3-D Figures

Lateral Area – does not include base areas (ex: toilet paper roll, b-day party hat)

Surface Area – does include base areas (ex: soda can, closed box)

Volume – amount filled inside 3-D figure (ex: soda in a can, helium in a balloon)

G.14 Proportions in similar figures

Scale Factor $a:b$

Perimeter ratio $a:b$

Any Area ratio $a^2:b^2$

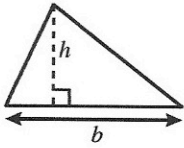
Volume ratio $a^3:b^3$

Think about the measurement units for perimeter, area, and volume to help you remember the power of the ratio

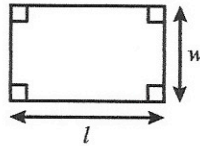
Geometry Formula Sheet

2016 Mathematics Standards of Learning

Geometric Formulas

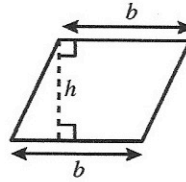


$$A = \frac{1}{2}bh$$

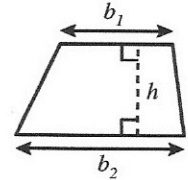


$$p = 2l + 2w$$

$$A = lw$$

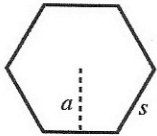


$$A = bh$$



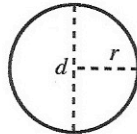
$$A = \frac{1}{2}h(b_1 + b_2)$$

Regular Hexagon



$$A = \frac{3\sqrt{3}}{2}s^2$$

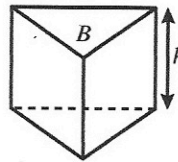
$$A = \frac{1}{2}pa$$



$$C = 2\pi r$$

$$C = \pi d$$

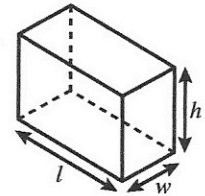
$$A = \pi r^2$$



$$V = Bh$$

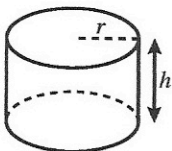
$$L.A. = hp$$

$$S.A. = hp + 2B$$



$$V = lwh$$

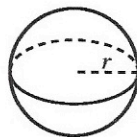
$$S.A. = 2lw + 2lh + 2wh$$



$$V = \pi r^2 h$$

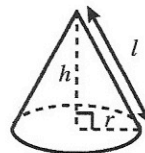
$$L.A. = 2\pi r h$$

$$S.A. = 2\pi r^2 + 2\pi r h$$



$$V = \frac{4}{3}\pi r^3$$

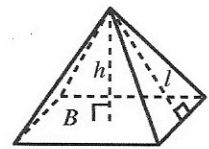
$$S.A. = 4\pi r^2$$



$$V = \frac{1}{3}\pi r^2 h$$

$$L.A. = \pi r l$$

$$S.A. = \pi r^2 + \pi r l$$



$$V = \frac{1}{3}Bh$$

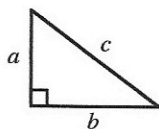
$$L.A. = \frac{1}{2}lp$$

$$S.A. = \frac{1}{2}lp + B$$

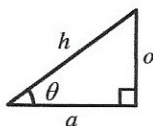
Geometry Formula Sheet

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Geometric Formulas



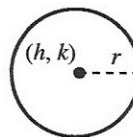
$$a^2 + b^2 = c^2$$



$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$



$$(x - h)^2 + (y - k)^2 = r^2$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0$$

Geometric Symbols

Example	Meaning
$m\angle A$	measure of angle A
AB	length of line segment AB
\overrightarrow{AB}	ray AB
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Line AB is parallel to line CD .
$\overline{AB} \perp \overline{CD}$	Line segment AB is perpendicular to line segment CD .
$\angle A \cong \angle B$	Angle A is congruent to angle B .
$\triangle ABC \sim \triangle DEF$	Triangle ABC is similar to triangle DEF .

Abbreviations

Area	A
Area of Base	B
Circumference	C
Lateral Area	$L.A.$
Perimeter	p
Surface Area	$S.A.$
Volume	V