EXERCISE SET 3 ANSWER KEY

No Calculator

1. **12** When the expression $(x - a)(x - b)(x - c)$ is fully distributed and simplified, it yields the expression $x^3 - (a + b + c)x^2 + (ab + bc + ac)x - abc$. If this is equivalent to $x^3 - 7x^2 + 16x - 12$ for all values of x, then all of the corresponding coefficients must be equal.

2. **7** See question 1.

3. **16** See question 1.

4. **7** If $x^2 - ax + 12 = 0$ when $x = 3$, then

$(3)^2 - 3a + 12 = 0$

Simplify: $21 - 3a = 0$
Add $3a$: $21 = 3a$
Divide by 3: $7 = a$

5. **4** As we saw in question 4, $a = 7$.

$x^2 - 7x + 12$

Factor: $(x - 3)(x - 4)$
Therefore, the zeros are 3 and 4.

6. **7/4 or 1.75** $x + y = 16$

Subtract x: $y = 16 - x$
Substitute: $16 - x = 4x^2 + 2$
Subtract 16, add x: $0 = 4x^2 + x - 14$
Factor: $0 = (4x - 7)(x + 2)$
Therefore, $x = -2$ or 7/4, but if x must be positive, it equals 7/4.

7. **B** The graph of the given equation is a circle centered at the origin with a radius of 3. Therefore, the horizontal line at $y = -3$ just intersects it at $(0, -3)$. You can also substitute $y = -3$ into the original equation and verify that it gives exactly one solution.

8. **C** Just notice the sign of each factor for each input:
$g(0.5) = (-)(+)(-)(-) =$ negative
$g(1.5) = (-)(+)(-)(-) =$ negative
$g(2.5) = (-)(+)(+)(-) =$ positive
$g(3.5) = (-)(+)(+)(+) =$ negative
Since (C) is the only option that yields a positive value, it is the greatest.

9. **A** $2x^2 + ax + b$

If $x = 5$ is a zero: $2(5)^2 + 5a + b = 0$
Subtract 50: $5a + b = -50$
If $x = -1$ is a zero: $2(-1)^2 + a(-1) + b = 0$
Subtract 2: $-a + b = -2$
Multiply by -1: $a - b = 2$
Add equations: $6a = -48$
Divide by 6: $a = -8$
Substitute $a = -8$: $-8 - b = 2$
Add 8: $-b = 10$
Multiply by -1: $b = -10$
Therefore, $a + b = -8 + -10 = -18$.

10. **D**

Substitute (2, 12): $12 = a(2)^4 + b(2)$
Simplify: $16a + 2b = 12$
Substitute (−2, 4): $4 = a(-2)^4 + b(-2)$
Simplify: $16a - 2b = 4$
Add two equations: $32a = 16$
Divide by 32: $a = ½$
Substitute: $16(1/2) + 2b = 12$
Subtract 8: $2b = 4$
Divide by 2: $b = 2$
Therefore, $a + b = 2.5$.

11. **A** Use the Zero Product Property. The factor $(x^2 + 1)$ cannot be zero for any value of x, $(x^3 - 1)$ is zero when $x = 1$, and $(x + 2)$ is zero when $x = -2$. Therefore, there are only two distinct points in which this graph touches the x-axis.

Calculator

12. **5** Substitute $y = 25$: $x^2 + 25 = 10x$

Subtract $10x$: $x^2 - 10x + 25 = 0$
Factor: $(x - 5)(x - 5) = 0$
Use Zero Product Property: $x = 5$

13. **108** If $x = 4$ is a zero: $2(4)^3 - 5(4) - a = 0$

Simplify: $108 - a = 0$
Add a: $108 = a$

14. **11** $x^4 - 9x^3 - 22x^2 = 0$

Divide by x^2: $x^2 - 9x - 22 = 0$
Factor: $(x - 11)(x + 2) = 0$
Use Zero Product Property: $x = 11$ or -2

15. **8** $y = (x^2)(x^2 + x - 72)(x - d)$

Factor: $y = (x^2)(x + 9)(x - 8)(x - d)$
By the Zero Property, the zeros are $x = 0$, -9, 8, or d. Since d is positive, but there can only be one positive zero, $d = 8$.

16. **12** $y = 2x^2 + 18$

Substitute $y = ax$: $ax = 2x^2 + 18$
Subtract ax: $0 = 2x^2 - ax + 18$
Divide by 2: $0 = x^2 - \frac{a}{2}x + 9$

If the graphs intersect in only one point, the system must have only one solution, so this quadratic must be a "perfect square trinomial" as discussed in Lesson 4.

$$x^2 - \frac{a}{2}x + 9 = x^2 - 2bx + b^2$$

Equate coefficients: $b^2 = 9$

$2b = a/2$

The only positive solution to this system is $b = 3$ and $a = 12$.

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17. **144** $4a^2 - 5b = 16$

$3a^2 - 5b = 7$

Subtract equations: $a^2 = 9$

Substitute $a^2 = 9$: $3(9) - 5b = 7$

Subtract 27: $-5b = -20$

Divide by -5: $b = 4$

Therefore, $a^2b^2 = 9(4)^2 = 144$.

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18. **B** In order for the product of three numbers to be negative, either all three numbers must be negative or exactly one must be negative and the others positive. Since n must be a positive integer, $n - 1$ cannot be negative, and so there must be two positive factors and one negative. The only integers that yield this result are the integers from 10 to 16, inclusive, which is a total of seven integers.

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19. **C** $x^2 + 2y^2 = 44$

Substitute $y^2 = x - 2$: $x^2 + 2(x - 2) = 44$

Distribute: $x^2 + 2x - 4 = 44$

Subtract 44: $x^2 + 2x - 48 = 0$

Factor: $(x - 6)(x + 8) = 0$

This seems to imply that the x-coordinate of the point of intersection could be either 6 or -8, both of which are choices. Can they both be correct? No: if we substitute $x = -8$ into either equation, we get no solution, because y^2 cannot equal -8. Therefore, the correct answer is (C) 6, and the points of intersection are (6, 2) and (6, −2).

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20. **C** $2m^2 + 2n = 14$

$m^2 + 2n = 10$

Subtract equations: $m^2 = 4$

Take square root: $m = \pm 2$

Substitute $m^2 = 4$: $4 + 2n = 10$

Subtract 4: $2n = 6$

Divide by 2: $n = 3$

Therefore, $m + n = -2 + 3 = 1$ or $2 + 3 = 5$.

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21. **A** Use the Zero Product Property. $(x^2 - 4)$ equals 0 if x is 2 or -2, $(x - 4)$ equals 0 if x is 4, and $(x^2 + 4)$ cannot equal 0. Therefore, there are exactly three distinct zeros.

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22. **C** $f(2.5) = a(2.5 + 2)(2.5 - a)(2.5 - 8)$

Simplify: $(-24.75)(a)(2.5 - a)$

This product can only be negative if a and $(2.5 - a)$ have the same sign, which is only true for (C) $a = 2$.