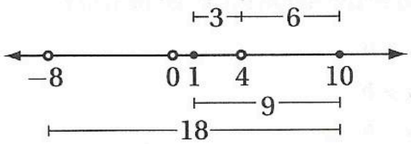
EXERCISE SET 4 ANSWER KEY

No Calculator

1. **4** It is helpful to plot these values on the number line and think:



The distance between 1 and 10 is 9, so clearly the number that is 9 more units to the left of 1, namely −8, is twice as far from 10 as it is from 1. However, this is a negative number so it can't be our answer. There is one other number that is twice as far from 10 as it is from 1: the number that is 1/3 the distance from 1 to 10. This number is 4, which is 3 units from 1 and 6 units from 10.

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2. **16** From the Distance Formula,

$$(2-14)^2+(a-b)^2=20^2$$

Simplify: $144+(a-b)^2=400$

Subtract 144: $(a-b)^2=256$

Square root: $|a-b|=16$

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3. **8** $0<\frac{4}{n}<\frac{5}{9}$

Since n must be positive for this statement to be true, we can multiply by $9n$ without having to "swap" the inequality symbols:

$$0<36<5n$$

Divide by 5: $0<7.2<n$

Therefore, the smallest integer value of n is 8.

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4. **½ or .5** Two numbers, a and b, have the same absolute value only if they are equal or opposites. Clearly $x+4$ and $x-5$ cannot be equal, since $x-5$ is 9 less than $x+4$. Therefore they must be opposites.

$$x+4=-(x-5)$$

Distribute: $x+4=-x+5$

Add x: $2x+4=5$

Subtract 4: $2x=1$

Divide by 2: $x=1/2$

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5. **10** $-\frac{n}{21}>-\frac{1}{2}$

Multiply by −42 and "swap:" $2n<21$

Divide by 2: $n<10.5$

Therefore, the greatest possible integer value of n is 10.

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6. **3** $3b\geq 7.5$

Divide by 3: $b\geq 2.5$

$$\frac{1}{b}>\frac{3}{11}$$

Since b is greater than or equal to 2.5, it is positive, so we can multiply both sides by $11b$ without "swapping" the inequality:

$$11>3b$$

Divide by 3: $3.67>b$

The only integer between 2.5 and 3.67 is 3.

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7. **3/2 or 1.5** $(b+2)^2=(b-5)^2$

FOIL: $b^2+4b+4=b^2-10b+25$

Subtract b^2: $4b+4=-10b+25$

Add $10b$: $14b+4=25$

Subtract 4: $14b=21$

Divide by 14: $b=1.5$

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8. **A** $-4<2x\leq 2$

Divide by 2: $-2<x\leq 1$

which is equivalent to $-2<x$ and $x\leq 1$.

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9. **A** The profit is the revenue minus the cost: $65n-(20{,}000+10n)=55n-20{,}000$.

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10. **C** If Colin can read a maximum of 25 pages an hour, then in h hours he can read a maximum of $25h$ pages. If he has p pages left in a 250-page book, he has read $250-p$ pages. Since it has taken him h hours to read these $250-p$ pages, $250-p\leq 25h$.

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11. **C** $|x-10|>4|x-40|$

It helps to sketch the number line and divide is into three sections: the numbers less than 10, the numbers between 10 and 40, and the numbers greater than 40.

CASE 1: $x<10$. It should be clear that all numbers less than 10 are closer to 10 than they are to 40, so this set contains no solutions.

CASE 2: $10<x\leq 40$. If x is between 10 and 40, $x-10$ is positive and $x-40$ is negative, so $|x-10|=x-10$ and $|x-40|=-(x-40)$.

$$|x-10|>4|x-40|$$

Substitute: $x-10>-4(x-40)$

Distribute: $x-10>-4x+160$

Add $4x$: $5x-10>160$

Add 10: $5x>170$

Divide by 5: $x>34$

So this gives us $34 < x \le 40$.

CASE 3: $x > 40$. If x is greater than 40, then both $x - 10$ and $x - 40$ are positive, so $|x - 10| = x - 10$ and $|x - 40| = x - 40$.

	$\lvert x - 10\rvert > 4\lvert x - 40\rvert$
Substitute:	$x - 10 > 4(x - 40)$
Distribute:	$x - 10 > 4x - 160$
Add 10:	$x > 4x - 150$
Subtract $4x$:	$-3x > -150$
Divide by −3 and "swap:"	$x < 50$

So this gives us $40 < x < 50$. When we combine this with the solutions from CASE 2, we get $34 < x < 50$.

Calculator

12. **2** If $|a - 5| = 7$, then either $a - 5 = 7$ or $a - 5 = -7$, so either $a = 12$ or $a = -2$. Since $a < 0$, a must be -2, and $|-2| = 2$.

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13. **8** CASE 1: If $6 - 3n$ is positive, then

$\lvert 6 - 3n\rvert = 6 - 3n$, so	$16 > 6 - 3n > 19$
Subtract 6:	$10 > -3n > 13$
Divide by −3 and "swap:"	$-10/3 < n < -13/3$

But this contradicts the fact that n is positive.

CASE 2: If $6 - 3n$ is negative, then

$\lvert 6 - 3n\rvert = -(6 - 3n)$, so	$16 > -(6 - 3n) > 19$
Distribute:	$16 > -6 + 3n > 19$
Add 6:	$22 > 3n > 25$
Divide by 3:	$7.33 > n > 8.33$

And the only integer in this range is $n = 8$.

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14. **7** $20 - 2n > 5$

Subtract 20:	$-2n > -15$
Divide by −2 and "swap:"	$n < 7.5$
	$\frac{2n}{3} > 4$
Multiply by 3:	$2n > 12$
Divide by 2:	$n > 6$

Since n must be an integer between 6 and 7.5, $n = 7$.

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15. **4.75** The distance from 3 to −1.5 is $|3-(-1.5)| = 4.5$. Therefore the two numbers that are 4.5 away from 9.25 are $9.25 + 4.5 = 13.75$ and $9.25 - 4.5 = 4.75$.

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16. **½ or .5** If the equation is true for all values of x, let's choose a convenient value for x, like $x = 1$.

	$\lvert 2x + 1\rvert = 2\lvert k - x\rvert$
Substitute $x = 1$:	$\lvert 2(1) + 1\rvert = 2\lvert k - 1\rvert$
Simplify:	$3 = 2\lvert k - 1\rvert$
Divide by 2:	$1.5 = \lvert k - 1\rvert$
Therefore	$\pm 1.5 = k - 1$
Add 1:	$k = 2.5$ or -0.5
Now try $x = 0$:	$\lvert 2(0) + 1\rvert = 2\lvert k - 0\rvert$
Simplify:	$1 = 2\lvert k\rvert$
Divide by 2:	$0.5 = \lvert k\rvert$
Therefore	$\pm 0.5 = k$

Therefore, $k = -0.5$ and so $|k| = |-0.5| = 0.5$.

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17. **C** Recall that the expression $|x - 2|$ means "the distance from x to 2," so the statement $|x - 2| < 1$ means "The distance from x to 2 is less than 1." Therefore, the solution set is all of the numbers that are less than 1 unit away from 2, which are all the numbers between 1 and 3.

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18. **C** $\frac{a+b}{2} > \frac{c+2b}{2}$

Multiply by 2:	$a + b > c + 2b$
Subtract b:	$a > c + b$

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19. **B** The formal translation of this statement is $|x - 1| > |x - 3|$, which we can solve algebraically by considering three cases: (I) $x \le 1$, (II) $1 < x \le 3$, and (III) $x > 3$, but it is probably easier to just graph the number line and notice that the midpoint between 1 and 3, that is, 2, is the point at which the distance to 1 and the distance to 3 are equal. Therefore, the points that are farther from 1 than from 3 are simply the points to the right of this midpoint, or $x > 2$.

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20. **B** $4x^2 \ge 9$

Take square root:	$\lvert 2x\rvert \ge 3$
If $x > 0$:	$2x \ge 3$
Divide by 2:	$x \ge 1.5$
If $x < 0$:	$2x \le -3$
Divide by 2:	$x \le -1.5$

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21. **D** Notice that the midpoint of the segment shown is 3, and the graph shows all points that are less than 3 units in either direction. Therefore, $|x - 3| < 3$.

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22. **B** (A) is untrue if $x = 0$, (C) is untrue for $x = -2$, and (D) is untrue if $x = 0.5$. But (B) is true for any real number.