EXERCISE SET 1: GEOMETRY ANSWER KEY

No Calculator

1. **16/9 or 1.77 or 1.78** If we define x as the length of $\overline{QN}$, then the length of one side of the square is $2x$, and so the area of square $MNOP$ is $(2x)(2x) = 4x^2$. To find this value, we can apply the Pythagorean Theorem to right triangle QNO:

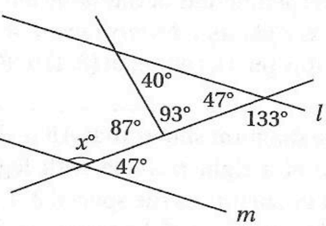
$$x^2 + (2x)^2 = \left(\frac{\sqrt{20}}{3}\right)^2$$

Simplify: $$5x^2 = \frac{20}{9}$$

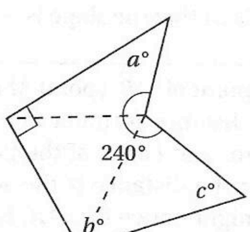
Divide by 5: $$x^2 = \frac{20}{45} = \frac{4}{9}$$

Multiply by 4: $$4x^2 = \frac{16}{9} = 1.77 \text{ or } 1.78$$

2. **133** The key is to notice simple relationships between angles until we get around to x.

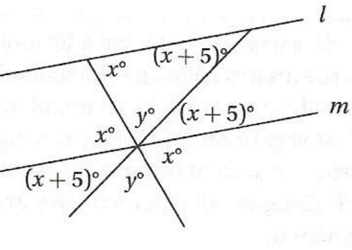


3. **210** Draw three lines as shown:



Since the polygon divides into 3 triangles, the sum of its internal angles is $(3)(180°) = 540°$. Therefore $a + b + c + 240 + 90 = 540$, and so $a + b + c = 210$.

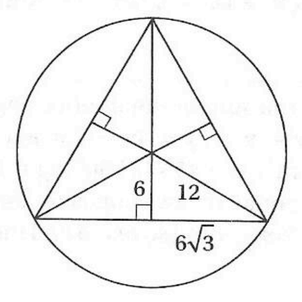
4. **C** Using the Crossed Lines Theorem and the Parallel Lines Theorem, we can mark up the diagram like this:



This shows that $x + y + x + 5 = 180$, and so $y = 175 - 2x$.

5. **B** The midpoint of the top segment is $\left(\frac{3+12}{2}, \frac{13+13}{2}\right) = \left(\frac{15}{2}, 13\right)$, and the midpoint of the bottom segment is $\left(\frac{3+18}{2}, \frac{5+5}{2}\right) = \left(\frac{21}{2}, 5\right)$, therefore, the distance between them is $\sqrt{\left(\frac{21}{2} - \frac{15}{2}\right)^2 + (13-5)^2} = \sqrt{3^2 + 8^2} = \sqrt{73}$

6. **C** To solve this problem we must draw a diagram and find the relationship between the radius of the circle and the sides of the triangle. By the Isosceles Triangle Theorem, if all three sides of a triangle are congruent, then all three angles must be congruent. Since these angles also must have a sum of 180°, they must each be 60°. If we draw the bisectors of each of these angles, we divide the triangle into six smaller triangles. These smaller triangles are congruent 30°-60°-90° triangles, as shown here:

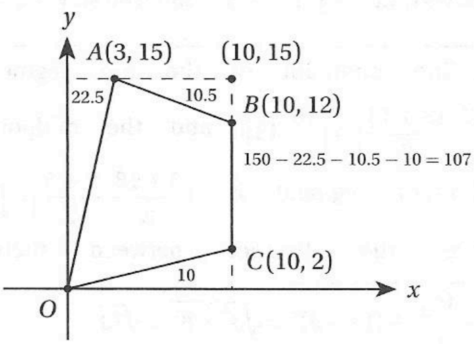


Since the circumference of the circle $(2\pi r)$ is 24π, its radius is 12. Since each of the hypotenuses of our right triangles is also a radius of the circle, we can find all of the sides of these triangles using the 30°-60°-90° relationships. Each side of the equilateral triangle is therefore $2\left(6\sqrt{3}\right) = 12\sqrt{3}$, and its perimeter is therefore $2\left(12\sqrt{3}\right) = 36\sqrt{3}$.

Calculator

7. **43** Using the distance formula, we can calculate the lengths of each segment. $OA = \sqrt{234} \approx 15.30$, $AB = \sqrt{58} \approx 7.61$, $BC = 10$, and $OC = \sqrt{104} \approx 10.20$. Therefore, the perimeter is approximately $15.30 + 7.61 + 10 + 10.20 = 43.11$, which rounds to 43.

8. **107** Since we do not have a formula that directly calculates the area of such an odd-shaped quadrilateral, we must analyze its area in terms of simpler shapes. The simplest way to do this is by drawing a box around it. This turns the area of interest into a rectangle minus three right triangles, all of which have areas that can be easily calculated.

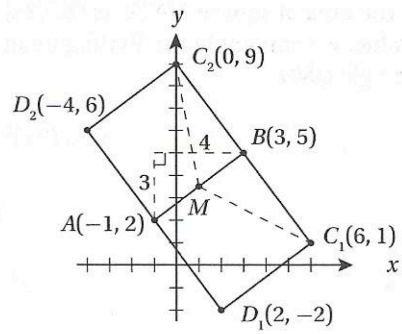


9. **6/5 or 1.2** The midpoint of $\overline{OA}$ is (1.5, 7.5) and the midpoint of $\overline{AB}$ is (6.5, 13.5); therefore, the slope of the segment between them is 6/5.

10. **36** If point *K* is on the same horizontal line as (2, 1), it must have a *y*-coordinate of 1, and if it is on the same vertical line as (11, 13), it must have an *x*-coordinate of 11. Therefore, *K* is the point (11, 1), and so $HK = 9$, $JK = 12$, and $HJ = \sqrt{9^2 + 12^2} = \sqrt{225} = 15$. Notice that it is a 3-4-5 triangle!

11. **C** Since the sum of the interior angles of any triangle is 180°, $y + y + 2y = 4y = 180$, and therefore $y = 45$. Therefore, this is a 45°-45°-90° right triangle. Since two angles are equal, the two opposite sides must also be equal, so $3m = 2m + 5$ and so $m = 5$ and the two legs each have measure 15. Using the Pythagorean Theorem or the 45°-45°-90° shortcut, we can see that $x = 15\sqrt{2}$.

12. **A** The key to questions 12 through 15 is a good diagram in the *xy*-plane that represents the given information:



If *ABCD* is a square, then the points *A*, *B*, *C*, and *D* must appear *in that order* around the square. Notice that to get from point *A* to point *B*, we must move 4 units to the right and 3 units up. This means that, in order to get to point *C* along a perpendicular of the same length, we must go either *3 units right and 4 units down,* or *3 units left and 4 units up.* This puts us either at (6, 1) or (0, 9).

13. **A** The diagram shows that *AB* is the length of the hypotenuse of a right triangle with legs 3 and 4. You should recognize this as the special 3-4-5 right triangle. If $AB = 5$, then the area of the square is $5^2 = 25$.

14. **A** Notice that the slope of $\overline{BC}$ is the same regardless of which option we choose for *C*. In either case, the slope formula tells us that the slope is −4/3.

15. **D** The midpoint of $\overline{AB}$ (point *M* above) is (1, 3.5). We can use the distance formula to find the distance between this point and either of the possible locations of *C*. (Notice that the distance is the same either way.) Alternately, we might notice that *MC* is the hypotenuse of a right triangle with legs 5 and 2.5. Either way, we get a value of $\frac{\sqrt{125}}{2}$.