EXERCISE SET 4: COMPLEX NUMBERS ANSWER KEY

No Calculator

1. **13** $(1 + 2i)(3 - 4i)$

FOIL: $(1)(3) + (1)(-4i) + (2i)(3) + (2i)(-4i)$

Simplify: $3 - 4i + 6i - 8i^2$

Substitute $i^2 = -1$: $3 - 4i + 6i - 8(-1)$

Combine like terms: $11 + 2i$

Therefore, $a = 11$ and $b = 2$, so $a + b = 13$.

2. **7/5 or 1.4** $\frac{4+i}{2-i}$

Multiply conjugate: $\frac{(4+i)(2+i)}{(2-i)(2+i)}$

FOIL: $\frac{8+4i+2i+i^2}{4+2i-2i-i^2}$

Substitute $i^2 = -1$: $\frac{8+4i+2i-1}{4+2i-2i+1}$

Combine like terms: $\frac{7+6i}{5}$

Distribute division: $\frac{7}{5}+\frac{6}{5}i$

3. **9** $(b + i)^2$

FOIL: $(b + i)(b + i) = b^2 + bi + bi + i^2$

Substitute $i^2 = -1$: $b^2 + bi + bi - 1$

Combine like terms: $(b^2 - 1) + 2bi$

Since this must equal $80 + 18i$, we can find b by solving either $b^2 - 1 = 80$ or $2b = 18$. The solution to both equations is $b = 9$.

4. **15** The equation we are given is a quadratic equation in which $a = 1$, $b = -2$, and $c = 15$. Therefore, we can use the quadratic formula:

Quadratic Formula: $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substitute: $\frac{2\pm\sqrt{(-2)^2-4(1)(15)}}{2(1)}$

Simplify: $\frac{2\pm\sqrt{-56}}{2}$

Simplify: $\frac{2\pm 2i\sqrt{14}}{2}$

Distribute division: $1\pm i\sqrt{14}$

Therefore, $a = 1$ and $b = 14$, so $a + b = 15$.

5. **B** $\frac{1}{(1+i)^2}$

FOIL: $\frac{1}{(1+i)(1+i)}=\frac{1}{1+i+i+i^2}$

Substitute $i^2 = -1$: $\frac{1}{1+i+i+(-1)}$

Simplify: $\frac{1}{2i}$

Multiply by i/i: $\frac{i}{2i^2}$

Substitute $i^2 = -1$: $\frac{i}{-2}=-\frac{1}{2}i$

6. **C** $(2 + 2i)^2$

FOIL: $(2 + 2i)(2 + 2i) = 4 + 4i + 4i + 4i^2$

Substitute $i^2 = -1$: $4 + 8i - 4 = 8i$

7. **D** $B(3 + i) = 3 - i$

Divide by $3 + i$: $B=\frac{3-i}{3+i}$

FOIL: $B=\frac{9-3i-3i+i^2}{9-3i+3i-i^2}$

Substitute $i^2 = -1$: $B=\frac{9-3i-3i+(-1)}{9-3i+3i-(-1)}$

Simplify: $B=\frac{8-6i}{10}=\frac{4-3i}{5}$

Distribute division: $B=\frac{4}{5}-\frac{3}{5}i$

8. **B** $x^2 + kx = -6$

Add 6: $x^2 + kx + 6 = 0$

Substitute $x=1-i\sqrt{5}$: $\left(1-i\sqrt{5}\right)^2+k\left(1-i\sqrt{5}\right)+6=0$

FOIL: $\left(1-2i\sqrt{5}+5i^2\right)+k\left(1-i\sqrt{5}\right)+6=0$

Simplify: $\left(-4-2i\sqrt{5}\right)+k\left(1-i\sqrt{5}\right)+6=0$

Distribute: $-4-2i\sqrt{5}+k-ik\sqrt{5}+6=0$

Collect terms: $(2+k)-\left(2\sqrt{5}+k\sqrt{5}\right)i=0$

Therefore, both $2 + k = 0$ and $2\sqrt{5}+k\sqrt{5}=0$. Solving either equation gives $k = -2$.

9. **B** As we discussed in Lesson 10, the powers of i are "cyclical," and $i^m = -i$ if and only if m is 3 more than a multiple of 4. The only number among the choices that is not 3 more than a multiple of 4 is (B) 18.