MATH ANALYSIS
CURRICULUM GUIDE

Overview and Scope & Sequence

Loudoun County Public Schools
2017-2018

(Additional curriculum information and resources for teachers can be accessed through CMS and VISION)

Mathematical Analysis Semester Overview

Semester 1	Semester 2
Functions MA.1 MA.2 MA.3 **Exponents & Logs** MA.9 ***MA.2 – EKS- End Behavior Exponential and Log Functions (2016) – (2009 – MA.9)*** **Conic Sections** MA.8 ***MA.6 EKS – Graph conic sections from equations written in vertex or standard form (2016) – (2009 – MA.8)*** **Trigonometry** T.6 T.7 T.5 T.8 MA.13 ***MA.13 EKS – Classify types of discontinuity; proving continuity at a point, using the definition of limits (2016)*** **Matrices** MA.14 ***MA.11 EKS – Verify two matrices are inverses using matrix multiplication (2016) – (2009 – MA.14)*** **Polar Graphing** MA.10	**Vectors** MA.11 **Parametric Equations** MA.12 **Sequences, Series&** MA.5 **Mathematical Induction** MA.6 MA.4 ***MA.13 EKS – Derive the formulas associated with arithmetic and geometric sequences and series (2016) – (2009-MA.5)*** **Limits** MA.7 MA.3 LCPSCALC 1.1 LCPSCALC 1.2 LCPSCALC 1.3 **Differentiation** LCPSCALC 2.1 LCPSCALC 2.2 LCPSCALC 2.3 LCPSCALC 2.4 LCPSCALC 2.5 LCPSCALC 2.6 **Applications** LPCPSCALC 3.1-3.6
46 blocks	**44 blocks**

Scope and Sequence

Number of Blocks	Topics and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
7 blocks **(including assessment)**	**Functions:** • Graphing—families of functions • Transformations • Domain, range, intercepts, • Odd/even, increasing/ decreasing, • Maximum/minimum, continuity • Compositions, inverse functions • Polynomial functions: end behavior, • Rational Functions: vertical, horizontal, oblique asymptotes, discontinuities	**MA.1** Identify a polynomial function, given an equation or graph. Identify rational functions, given an equation or graph. Identify domain, range, zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points, given a graph of a function. Sketch the graph of a polynomial function. Sketch the graph of a rational function. Investigate and verify characteristics of a polynomial or rational function, using a graphing calculator. The graphs of polynomial and rational functions can be determined by exploring characteristics and components of the functions. **MA.2** Find the composition of functions. Find the inverse of a function algebraically and graphically. Determine the domain and range of the composite functions. Determine the domain and range of the inverse of a function. Verify the accuracy of sketches of functions, using a graphing utility. In composition of functions, a function serves as input for another function. A graph of a function and its inverse are symmetric about the line $y = x$. $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$	**Emphasize rational functions **Emphasize calculus vocab **review/teach partial fraction **Links to Websites:** Precalculus: Inverses of Functions by Texas Instruments Angry Bird Parabola Project Birthday Polynomial Rational Function Project Building Connections Shrinking Candles, Running Water, Folding Boxes

Number of Blocks	Topics and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
(cont. from previous)	(cont. from previous)	**MA.3** Describe continuity of a function. Investigate the continuity of absolute value, step, rational, and piece-wise-defined functions. Use transformations to sketch absolute value, step, and rational functions. Verify the accuracy of sketches of functions, using a graphing utility. Continuous and discontinuous functions can be identified by their equations or graphs.	(cont. from previous) **add f(x)= abs(x)/x ** vocab of removable and non-removable
7 blocks **(including assessment)**	Exponential and logarithmic functions: Graphing, Properties, Solving Equations, Law of Exponential Growth/Decay, Compound Interest, Logistics	**MA.9** Identify exponential functions from an equation or a graph. Identify logarithmic functions from an equation or a graph. Define e, and know its approximate value. Write logarithmic equations in exponential form and vice versa. Identify common and natural logarithms. Use laws of exponents and logarithms to solve equations and simplify expressions. Model real-world problems, using exponential and logarithmic functions. Graph exponential and logarithmic functions, using a graphing utility, and identify asymptotes, intercepts, domain, and range. Exponential and logarithmic functions are inverse functions. Some examples of appropriate models or situations for exponential and logarithmic functions are: – Population growth;Compound interest; – Depreciation/appreciation;Richter scale; and – Radioactive decay.	**Emphasize Solving Equations **Emphasize Properties **Include Applications ** Emphasize e^x **Links to Websites:** Precalculus: Accelerated Returnsby Texas Instruments Precalculus: Can You Hear Me Now?by Texas Instruments Shrinking Candles, Running Water, Folding Boxes

Semester 1

Number of Blocks	Topics and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
4 blocks **(including assessment)**	**Conic Sections:** • Graphs • Identifying/and classifying conic sections • General and standard Form, Transformations	**MA.8** Given a translation or rotation matrix, find an equation for the transformed function or conic section. Investigate and verify graphs of transformed conic sections, using a graphing utility. Matrices can be used to represent transformations of figures in the plane.	**Review of completing the square is suggested **Include solving systems http://math2.org/math/algebra/conics.htm to see pictures of how the conic sections are formed when a plane cuts through a cone. This website will also have the equations for the conic sections. To see some awesome pictures of conics and to read about the history of conics, visit http://xahlee.org/SpecialPlaneCurves_dir/ConicSections_dir/conicSections.html Visit this website for further lessons on how to work conic section problems. There are also some real-life applications to be found here. http://jwilson.coe.uga.edu/emt669/student.Folders/Jones.June/conics/conics.html

Semester 1

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
17 total blocks for this entire unit **(including assessment)**	**Trigonometry:** • Graphing trigonometric functions • Amplitude • Period • Phase shift • Vertical Shift • Asymptotes **(3 blocks graphing)**	**T.6** The student, given one of the six trigonometric functions in standard form, will a) state the domain and the range of the function; b) determine the amplitude, period, phase shift, vertical shift, and asymptotes; c) sketch the graph of the function by using transformations for at least a two-period interval; and d)investigate the effect of changing the parameters in a trigonometric function on the graph of the function. Determine the amplitude, period, phase shift, and vertical shift of a trigonometric function from the equation of the function and from the graph of the function. Describe the effect of changing *A*, *B*, *C*, or *D* in the standard form of a trigonometric equation {e.g. $y = A\sin(Bx+C)+D$ or $y = A\cos(Bx+C)+D$} State the domain and the range of a function written in standard form {e.g. $y = A\sin(Bx+C)+D$ or $y = A\cos(Bx+C)+D$} Sketch the graph of a function written in standard form {e.g. $y = A\sin(Bx+C)+D$ or $y = A\cos(Bx+C)+D$} by using transformations for at least one period or one cycle. The domain and range of a trigonometric function determine the scales of the axes for the graph of the trigonometric function. The amplitude, period, phase shift, and vertical shift are important characteristics of the graph of a trigonometric function, and each has a specific purpose in applications using trigonometric equations. The graph of a trigonometric function can be used to display information about the periodic behavior of a real-world situation, such as wave motion or the motion of a Ferris wheel.	**Check with Alg II/Trig teacher to see how much content was covered due to excessive snow days **Unit Circle (T.1-T.5) covered in Alg 2/Trig. **Review graphs of six trig functions **Emphasize Phase Shift **Links to Websites:** Hands on Trig iOS - TRIGO Android - Trig Quizzer Graphing Trig Project Precalculus: Find That Sine by Texas Instruments Precalculus: Vertical and Phase Shifts by Texas Instruments

Semester 1

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
	Inverse Trigonometric Functions: • Graphing • Domain/Range • Evaluating **(3 blocks for Inverse Trig Functions)**	**T.7** The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included. Find the domain and range of the inverse trigfunctions. Use the restrictions on the domains of the inverse trigonometric functions in finding the values of the inverse trigonometric functions. Identify the graphs of the inverse trigonometric functions.	**Pythagorean Triples, Variable Sides, Composition with Trig. Functions **Understanding calculator interpretation **Links to Websites:** Desmos: Intro to Inverse Trig
	Trigonometric Properties/ Identities: Sum/difference Half angle, double angle Establishing identities Solving trigonometric equations **(5 blocks for Trig Properties/Identities)**	**T.5** The student will verify basic trigonometric identities and make substitutions, using the basic identities. Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include: – reciprocal identities; – Pythagorean identities; – sum and difference identities; – double-angle and half-angle identities; Trigonometric identities can be used to simplify trigonometric expressions, equations, or identities. Trigonometric identity substitutions can help solve trigequations, verify another identity, or simplify trig expressions. **T.8** Solve trigonometric equations with restricted domains algebraically and by using a graphing utility. Solve trigonometric equations with infinite solutions algebraically and by using a graphing utility. Check for reasonableness of results, and verify algebraic solutions, using a graphing utility.	**Focus on derivations of formulas **Continue focus on establishing identities throughout **Variety of forms of Trig Equations (quadratic, etc.) *A calculator can be used to find the solution of a trigonometric equation as the points of intersection of the graphs when one side of the equation is entered in the calculator as Y_1 and the other side is entered as Y_2.

		Solutions for trig equations will depend on the domains.	

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
	Triangle Trigonometry Applications: • Law of Sines • Law of Cosines **(3 blocks for Triangle Trig)**	**MA.13** Solve and create problems, using trigonometric functions. Solve and create problems, using the Pythagorean Theorem. Solve and create problems, using the Law of Sines and the Law of Cosines. Solve real-world problems using vectors. Real-world problems can be modeled using trigonometry and vectors.	**Review right triangle trig in a problem set (seen in Geometry) **Area of Triangles, as time permits **Links to Websites:** Exploration/Discovery Law of Sines Gizmos – Proving Triangles Congruent **(Choose "SSA". Discuss conditions for Counterexample.)**

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
3 blocks **(including assessment)**	**Matrices** • Basic matrix operations • Solving systems of equations	**MA.14** Add, subtract, and multiply matrices and multiply matrices by a scalar. Model problems with a system of no more than three linear equations. Express a system of linear equations as a matrix equation. Solve a matrix equation. Find the inverse of a matrix.** Verify the commutative and associative properties for matrix addition and multiplication. Matrices are convenient shorthand for solving systems of equations. Matrices can model a variety of linear systems. Solutions of a linear system are values that satisfy every equation in the system. Matrices can be used to model and solve real-world problems.	**Solving may include using inverse matrices, Cramer's Rule, and/or row echelon reduction **Find the inverse of a 2x2 matrix without a calculator. 3x3 calculator can be used. **Use a calculator to find the inverse of any matrix. **Links to Websites:** Precalculus: All Systems Go! by Texas Instruments Algebra II: Solving Systems Using Matrices by Texas Instruments

Semester 1

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
4 blocks **(including assessment)**	**Polar Coordinates and Graphs:** • Graphing in the polar plane • Converting between rectangular and polar coordinates • Common polar equations	**MA.10** Recognize polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation. Determine the effects of changes in the parameters of polar equations on the graph, using a graphing utility. Convert complex numbers from rectangular form to polar form and vice versa. Find the intersection of the graphs of two polar equations, using a graphing utility. The real number system is represented geometrically on the number line, and the complex number system is represented geometrically on the plane where $a + bi$ corresponds to the point (a, b) in the plane.	**Supplement text for graphs of polar equations
4 blocks	**Review and Common Assessment** **Refer to common review		

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
8 blocks total **(combine with parametric; include assessment)**	**Vectors:** • Basic Operations • Dot Product • Norm of a Vector • Unit Vector • Graphing • Properties • Proofs • Complex Numbers as Vector-perpendicular components (3 blocks)	**MA.11** Use vector notation. Perform the operations of addition, subtraction, scalar multiplication, and inner (dot) product on vectors. Graph vectors and resultant vectors. Express complex numbers in vector notation. Define *unit vector*, and find the unit vector in the same direction as a given vector. Identify properties of vector addition, scalar multiplication, and dot product. Find vector components. Find the norm (magnitude) of a vector. Use vectors in simple geometric proofs. Solve real-world problems using vectors. Every vector has an equal vector that has its initial point at the origin. The magnitude and direction of a vector with the origin as the initial point are completely determined by the coordinates of its terminal point.	**Supplement as necessary **Implement Real-World problems
	Parametric Equations: • Graphing • Converting to Rectangular (2 blocks)	**MA.12** Graph parametric equations, using a graphing utility. Use parametric equations to model motion over time. Determine solutions to parametric equations, using a graphing utility. Compare and contrast traditional solution methods with parametric methods. Parametric equations are used to express two dependent variables, x and y, in terms of an independent variable (parameter), t. Some curves cannot be represented as a function, $f(x)$. Parametric graphing enables the representation of these curves in terms of functions.	**Incorporate calculator ** Real-World Applications

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
8 total blocks for unit (combine with induction and binomial theorem; include assessment)	**Sequences and Series:** • Arithmetic/geometric sequences • Infinite--sums of convergent series (1 block)	**MA.5** Use and interpret the notation: $\sum$, n, nth, and a_n. Given the formula, find the nth term, a_n, for an arithmetic or geometric sequence. Given the formula, find the sum, S_n, if it exists, of an arithmetic or geometric series. Model and solve problems, using sequence and series information. Distinguish between a convergent and divergent series. Discuss convergent series in relation to the concept of a limit. Examination of infinite sequences and series may lead to a limiting process. Arithmetic sequences have a common difference between any two consecutive terms. Geometric sequences have a common factor between any two consecutive terms.	**Review finite series (covered in Alg 2/Trig) **Introduce idea of a limit **Links to Websites:** Arithmetic Sequence Discovery Precalculus: Infinite Geometric Series by Texas Instruments Recursive and Exponential Rules Algebra II: Arithmetic Sequences & Series by Texas Instruments
	Mathematical Induction (2 blocks)	**MA.6** Compare inductive and deductive reasoning. Prove formulas/statements, using mathematical induction. Mathematical induction is a method of proof that depends on a recursive process. Mathematical induction allows reasoning from specific true values of the variable to general values of the variable.	**Consider including examples extending beyond equalities
	Binomial Theorem: • Pascal's Triangle • Combinations\ (2 blocks)	**MA.4** Expand binomials having positive integral exponents. Use the Binomial Theorem, the formula for combinations, and Pascal's Triangle to expand binomials. The Binomial Theorem provides a formula for calculating the product $(a + b)^n$ for any positive integer n. Pascal's Triangle is a triangular array of binomial coefficients.	

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
9 blocks	**Limits:** • Numerically • Analytically • Graphically • Algebraically End Behavior Asymptotes One-sided Limits Definition of Continuity	**MA.7** Verify intuitive reasoning about the limit of a function, using a graphing utility. Find the limit of a function algebraically, and verify with a graphing utility. Find the limit of a function numerically, and verify with a graphing utility. Use limit notation when describing end behavior of a function. The limit of a function is the value approached by $f(x)$ as x approaches a given value or infinity. **MA.3** Continuous and discontinuous functions can be identified by their equations or graphs. **LCPSCalc.1.1 Limits of functions (including one-sided limits)** • An intuitive understanding of the limiting process • Calculating limits using algebra • Estimating limits from graphs or tables of data **LCPSCalc.1.2Asymptotic and unbounded behavior** • Understanding asymptotes in terms of graphical behavior • Describing asymptotic behavior in terms of limits involving infinity • Comparing relative magnitudes of functions and their rates of change (forexample, contrasting exponential growth, polynomial growth, and logarithmicgrowth) **LCPSCalc.1.3Continuityas a property of functions** • An intuitive understanding of continuity. (The function values can be made asclose as desired by taking sufficiently close values of the domain.) • Understanding continuity in terms of limits • Geometric understanding of graphs of continuous functions (IntermediateValue Theorem and Extreme Value Theorem)	**Collect Pre-Calculus books and distribute Calculus books **Include Section 3.5 limits at infinity **Links to Websites:** An intuitive guide to limits Why do we need limits? **Activities:** Desmos: Discovery Activity, Exploring Limits Graphically Introduction to Limits Activity IBL Continuity Intermediate Value Theorem Wacky Limits Worksheet Graphical Activity for Limits

			Making Limits Exist by Texas Instruments In this activity, students will graph piecewise functions and evaluate numerically and graphically the left-hand limit and the right-hand limit of the function as x approaches a given number, c. **Geogebra Applets** 1. Intuitive Notion of the Limit 2. Intuitive Notion of the Limit - One-Sided Limits 3. The Limit Laws 4. Continuity at a Point 5. An Important Trig Limit 6. Average and Instantaneous Rate of Change **Limits and Continuity** Hexaflexagon -- Limits Fractal Sheets -- Limits Limits -- Artistic Perspective Limits with Angles of Polygons Infinite Series Review of Limits I Have ... Who Has ... Cards -- Limits SUDOKU Puzzle with Limits Continuity -- Table Exploration

			Review of Limits and Continuity Limits -- Blueberry Pancakes Turvy for Limits & Continuity Proof of L'Hopital's Rule

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
15 blocks **(including assessment)**	**Differentiation:** • Tangent line problem • Limit of the difference quotient • Definition of derivatives • Differentiation rules • Graph analysis • Position • Velocity • Acceleration • Implicit differentiation • Related rates (Applications)	**LCPSCalc.2.1 Concept of the derivative** • Derivative presented graphically, numerically, and analytically • Derivative interpreted as an instantaneous rate of change • Derivative defined as the limit of the difference quotient • Relationship between differentiability and continuity **LCPSCalc.2.2Derivative at a point** • Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. • Tangent line to a curve at a point and local linear approximation • Instantaneous rate of change as the limit of average rate of change • Approximate rate of change from graphs and tables of values **LCPSCalc.2.3 Derivative as a function** • Corresponding characteristics of graphs of f and f' • Relationship between the increasing and decreasing behavior of f and the signof f' • The Mean Value Theorem and its geometric interpretation • Equations involving derivatives. Verbal descriptions are translated intoequations involving derivatives and vice versa. **LCPSCalc.2.4 Second derivatives** • Corresponding characteristics of the graphs of f, f', and f'' • Relationship between the concavity of f and the sign of f'' • Points of inflection as places where concavity changes	**Formal Definition (δ-ε) not required **Suggested: Exponential, Logarithms derivative rules after chain rule (Ch. 5) **Optional: Inverse Trig. derivative rules (Ch. 5) **Emphasize:** **The slope of a secant line is the average rate of change. **The slope of a tangent line is the instantaneous rate of change. **Real-World Applications for PVA, Related Rates **Refer to AP-Style Problems

Number of Blocks	Topic and Essential Questions	Standard(s) of Learning Essential Knowledge and Skills Essential Understandings	Additional Instructional Resources/Comments
(cont. from previous)	(cont. from previous)	**LCPSCalc.2.5Applications of derivatives** • Analysis of curves, including the notions of monotonicity and concavity • Optimization, both absolute (global) and relative (local) extrema • Modeling rates of change, including related rates problems • Use of implicit differentiation to find the derivative of an inverse function • Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration **LCPSCalc.2.6Computation of derivatives** • Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions • Derivative rules for sums, products, and quotients of functions • Chain rule and implicit differentiation	(cont. from previous). **Links to Websites:** Desmos: Sketchy Derivatives Desmos: Card Sort Derivative Match Derivative Matching Cards Desmos: Exploring the Graphs of Derivatives Desmos: Derivatives of a Polynomial Desmos: Graphing a Derivative Relating Derivative to average rate of change. Quizlet Derivative Rules Card Match Activity with descriptions **Limit Definition of Derivative Worksheet:** Guide Worksheet Derivatives of Trigonometric

			Functions **by Texas Instruments**
7 blocks	**Applications of Differentiation:** • Extrema • Rolle's Theorem • Mean Value Theorem • Curve Sketching • Optimization Linearization/ • Differentials	**Projects** Calculus Roller Coaster Project and rubric The Calculus of Rainbow Project Where to sit in a movie theater Project **Geogebra Applets:** 1. The Derivative at a Point 2. The Derivative as a Function 3. The Derivative of Elementary Functions 4. Try to Graph the Derivative Function 5. The Derivative of Exponential Functions 6. Identify the Derivative Function 7. Derivatives and Graph Transformations 8. Identify a Function and its First and Second Derivatives 9. Identify an Antiderivative Function 10. The Power Rule - Derivatives of Polynomial Functions 11. Intuitive Notion of the Chain Rule 12. Implicit Differentiation 13. Derivatives of Inverse Functions 14. First derivative test - Reconstruct f from its First Derivative 15. Second derivative test - Reconstruct f from its Second Derivative 16. Derivatives and the Shape of a Graph 17. Related Rates Problems: Oil Slick -- A Falling Ladder -- A Conical Tank -- Driving Car -- Lamppost -- Two Trains -- Searchlight -- Rocket Launch	** Introduce integration and incorporate AP-style practice **Links to Websites:** **Curve Sketching** **Curve Sketching Instructions** **The S-Curve** **Review Sheet on Curve Sketching** **The Seasons -- Curve sketching** **Methods for Finding Critical Points** **Determining Concavity** **Applications of the Derivative** **Class, Take Your Seats! (tangents and normals)** **Volume of a Box (Max/Min)** **Maxima / Minima Procedures** **Worksheet on Maxima / Minima Problems**

		18. Optimization Problems: Bending a Wire -- Rectangle Inscribed in a Parabola -- Three Pens -- Getting Power to an Island -- Another Wire Problem -- Function and Rectangle 1 -- Function and Rectangle 2 -- Triangle Circumscribing a Circle --	**Do Dogs Know Calculus? (Max / Min)** **Snell's Law (Max/Min)** **The Longest Home Run (Max / Min)** **Class, Take Your Seats! (Max/Min)** **Turvy with Applications of the Derivative** **Miniature Golf / Billiards (Max/Min)** **Maxima/Minima Review Problems** **Maxima/Minima Review Problems** **Cat Food Can Maxima/Minima Problem** **Letter from Carnation about Cat Food Problem** **The Math of the Soda Pop Can** **The Spider and the Fly (Max/Min puzzle)** **Winchester Population Problem**
4 blocks	**Review, Assessment, and Enrichment**		

Additional information about the Standards of Learning can be found in the

VDOE 2009 Curriculum Framework

VDOE 2016 Curriculum Framework

(click link above)

Additional information about math vocabulary can be found in the

VDOE Vocabulary Word Wall Cards

(click link above)