ADVANCED FUNCTIONS & MODELING CURRICULUM GUIDE

Overview

Loudoun County Public Schools
2017-2018

(Additional curriculum information and resources for teachers can be accessed through CMS and VISION)

Advanced Functions and Modeling Semester Overview

1st Semester			2nd Semester		
Topic	**Standard**	**Blocks**	**Topic**	**Standard**	**Blocks**
Function Characteristics	**AFM.4**	**4**	**Rational and Radical Functions**	**AFM.6**	**8**
Linear Functions	**AFM.1**	**8**	**Unit Circle Trigonometry**	**AFM.7** **AFM.8**	**7**
Quadratic Functions	**AFM.2**	**11**	**Trigonometric Functions and Inverses**	**AFM.9** **AFM.10**	**7**
Polynomial Functions	**AFM.3**	**11**	**Trigonometric Identities**	**AFM.11**	**8**
Exponential and Logarithmic Functions	**AFM.5**	**8**	**Trigonometric Equations**	**AFM.12**	**4**
			Applications of Trigonometry	**AFM.13**	**8**
Assessment, Enrichment and Remediation		**3**	**Assessment, Enrichment and Remediation**		**2**

Philosophy and Notes:

- **Teach necessary skills and problem solving techniques prior to starting lab/activity.**
- **Plan 2-3 class periods per lab**
- **Incorporate written summary for lab assignments.**

Resources:
Textbook: *College Algebra and Trigonometry*, Fifth Edition, Augmann, Barker, and Nation
Graphic Algebra, Key Curriculum Press, 1998
Mathematical Investigations Book Two, Dale Seymour Publications, 1992
NCSSM Distance Learning-website
Pacesetter Mathematics Precalculus through Modeling Volume 1, the College Board
Texas Instruments website
Math Smart, Jossey Bass, 2002
Zooming in on Precalculus: Explorations with Technology, D & S Marketing Systems, 2000

Scope and Sequence

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
4	**Function Family Characteristics** **AFM.4**The student will be find the domain, range, zeros, and inverse of a function, the value of a function for a given element in its domain, and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions and to solve real world problems. **<u>OBJECTIVES:</u>** The student will be able to: 1. Identify the domain, range, zeros, and inverse of a function presented algebraically or graphically. 2. Distinguish between relations and functions that are expressed algebraically and graphically. 3. Recognize restricted/discontinuous domains and ranges. 4. Use interchange of variables to find the inverse of a function. 5. Find the composition of two functions.	Conversion between Celsius and Fahrenheit- Mathematical Investigations Desmos– "Charge!"	**Activities:** • Temperature Scales- Pacesetter Mathematics • Unit Conversions- Mathematical Investigations • Kilometers/miles- Mathematical Investigations • ~~Cryptography – In coding & Decoding- Mathematical Investigations~~ • ~~Textbook Section 4.1 p. 366 #53-54~~ o ~~Using a Number Box Cipher Code breaking- Math Smart~~

Number		REQUIRED	Additional Instructional

Semester 1

of Blocks	Topics and Essential Questions	Critical Thinking Lessons	Resources
8	**Linear Functions** **AFM.1**The student will be able to identify, graph, and write linear functions and to apply the concepts of linear functions to real world models. **OBJECTIVES:** The student will be able to: 1. Recognize the solution of a linear equation is the zero of the function. 2. Fit linear functions to data by using algebra and technology. 3. Summarize and analyze results in application problems using reasoning/ problem solving techniques (i.e., What does the y-intercept represent in this problem? Explain how the answer does/ does not make sense in a real world situation? What other factors may influence your results when applied to a similar situation? 4. Recognize the special properties of parallel and perpendicular lines. 5. Graph piece-wise linear functions. 6. Graph absolute value equations in two variables and recognize them as piece-wise linear functions. 7. Find domain, range, end behavior, symmetry of linear functions.	Stack of Cups- Pacesetter Mathematics or Spaghetti Bridge- Mathematical Investigations	Activities: Mathematical Investigations ○ Penny Bridge- ○ Life Expectancy ○ Payoff Piece-wise Functions Textbook Section 2.2 ○ Postage ○ Price of gas ○ Electric meter ○ First class mail (p. 191) ○ Income tax (p. 192)

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
11	**Quadratic Functions** **AFM.2** The student will be able to identify, graph, and write quadratic functions and to apply the concepts of quadratic functions to	Desmos Quadratic Bundle	**Activities:** • Pennies in a Circle- Mathematical Investigations

	real-world models. **OBJECTIVES:** The student will be able to: 1. Graph quadratic equations by using the vertex and axis of symmetry, vertex/standard form, and transformations. 2. Select an appropriate strategy for solving a quadratic equation (factoring, completing the square, using the quadratic formula, or graphing). 3. Recognize the solution(s) of a quadratic equation is/are the zero(s) of the function. 4. Fit quadratic functions to data by using algebra and technology. 5. Solve a quadratic equation over the set of complex numbers. 6. Find domain, range, end behavior, symmetry of quadratic functions.	Pig Problem NCSSM AFM resources Extreme Punkin' Chunkin' Textbook Section 2.4 pp. 223-227 Textbook Section 2.7 pp. 263-267	• Number of points on circle vs Number of points drawn Mathematical Investigations ○ Projectile motion ○ Quadratic Functionsc2u4 Mathematical Investigations ○ Shot Put-Pacesetter Mathematics ○ Rectangular Enclosures ○ Fences- Pacesetter Mathematics ○ Holding Pen ○ Braking Distance-Graphic Algebra ○ Cost of operating a Ship-Graphic Algebra ○ Invention Kitchen Gadget - Cost versus profit of creating new invention- Graphic Algebra ○ Sydney Harbor Bridge example- Graphic Algebra ○ Falling objects- Graphic Algebra ○ TI website: "Old McDonald's Pigpen" ○ Parabola in Real Life Algebra II: Motorcycle Jump by Texas Instruments Football and Braking Distance

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
11	**Polynomial Functions** **AFM.3** The student will be able to identify, graph, and write polynomial functions and to apply the concepts of polynomial functions to real world models. **OBJECTIVES:** The student will be able to: 1. Recognize general shapes and end behavior of polynomial functions. 2. Graph polynomial functions by including relative extrema. 3. Recognize the solution(s) of a polynomial equation is/are the zero(s) of the function. 4. Fit polynomial functions to data by using algebra and technology. 5. Solve a polynomial equation over the set of complex numbers. Find the number of real versus the number of imaginary roots and describe how that affects the nature of the graph. Include Descartes' Rule of Signs 6. Write a polynomial function given zero(s). 7. Find domain, range, end behavior, symmetry of polynomial functions.	Oranges Stacked in a Square Based Pyramid- Mathematical Investigations or Barbie© Bungee- Mathematical Investigations Filling a Box with Diamonds! The Box Problem	**Activities:** ○ Textbook Section 3.2 ○ p. 302 #47-51 ○ Box problem (Maximize volume) ○ Maximize profit ○ TI website: "Explore End Behavior" (1.5) Polynomial Functions and Models- Mathematical Investigations Precalculus: Ride the Rollercoaster by Texas Instruments Precalculus: Going Back to Your Roots by Texas Instruments

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
8	**Exponential and Logarithmic Functions** **AFM.5** The student will be able to identify, graph, and write exponential and logarithmic functions and to apply the concepts of exponential and logarithmic functions to real world models. **OBJECTIVES:** The student will be able to: 1. Recognize general shapes of exponential and logarithmic functions. This should include common and natural logarithms. Find domain, range, and end behavior. 2. Graphically and algebraically recognize that exponential and logarithmic functions are inverses of each other. 3. Fit exponential and logarithmic functions to data by using algebra and technology. 4. Solve exponential and logarithmic equations by applying properties of exponents and logarithms. If time, investigate logistic growth models.	Desmos Exponential Bundle Textbook Exponential & Logarithmic Functions: Hurricane Fran or Harry Casey and the Pennsylvania Lottery- Pacesetter Mathematics Desmos - "iPhone 6s Opening Weekend Sales" "Card Sort: Modeling"	**Activities:** • M & M's/Penny Half-Life- Mathematical Investigations • Bacteria: Log Rhythms, or Half a Log is Better than None! Zooming in Precalculus Investigations • Buying a New Car Zooming in Precalculus Investigations • Credit Card Payoff Zooming in Precalculus Investigations • Loan Payoff Zooming in Precalculus Investigations ○ Thickness of Ozone Layer over time- Graphic Algebra • TI website: "Carbon dating" • TI website: "Analysis of a Bouncing Ball" • A Powerful Function- Pacesetter Mathematics • Population Growth- Pacesetter Mathematics • Textbook Section 4.5 pp. 415-418 Section 4.6 pp. 430-434 ○ Newton's Law of Cooling Movie Contract- Graphic Algebra Graph of Logarithm Equation Exponential Growth Equations and Graphs Modeling Orbital Debris Problems Smokey Bear Takes Algebra

Semester 1

3 blocksAssessments, Enrichment, and Remediation

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
8	**Rational and Radical Functions** **AFM.6** The student will be able to identify, graph, and write rational and radical functions and to apply the concepts of rational and radical functions to real world models. **OBJECTIVES:** The student will be able to: 1. Recognize general shapes of rational and radical functions. Find domain, range, and end behavior. This should include point, jump, and infinite discontinuities. 2. Add, subtract, multiply, and divide rational expressions whose denominators are monomials or polynomial expressions. 3. Simplify a rational expression with common monomial or binomial factors. 4. Recognize and simplify a complex fraction. 5. Solve equations containing rational expressions both algebraically and graphically. 6. Convert from radical notation to exponential notation, and vice versa. 7. Simplify radical expressions. 8. Add, subtract, multiply, and divide radical expressions. Do not require rationalizing the denominators. 9. Solve equations containing radical expression both algebraically and graphically. 10. Fit rational and radical functions to data by using algebra and technology.	Planning a Summer Camp- Pacesetter Mathematics Precalculus: Finding Extraneous Solutionsby Texas Instruments Algebra II: Domain & Range by Texas Instruments - Bell Ringer Lesson	**Activities:** ○ Weather Balloon Take-off- Graphic Algebra ○ Sharing Chocolates- Graphic Algebra ○ Area of a Farm Field- Graphic Algebra ○ Scuba Diving- Graphic Algebra • Wind Chill (Chilly Today, Hot Tamale) Zooming in Precalculus Investigations • Cost/Benefit Professor Rust with RUBRIC- Mathematical Investigations • Textbook Section 3.5 • pp. 342-344 ○ Average Cost #53-54 Graphing Rational Functions Lesson 14: Graphing Rational Functions - EngageNY Lesson 15: Transforming Rational Functions - EngageNY Precalculus: You Can't Get There From Hereby Texas Instruments Algebra II: Radical Transformationsby Texas Instruments Algebra II: Asymptotes and Zeros of Rational Functionsby Texas Instruments Algebra II: Roots of Radical Equations by Texas Instruments - Bell Ringer Lesson

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
3	**Unit Circle Trigonometry** **AFM.7** The student will be able to use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position whose terminal side contains a given point. Circular function definitions will be connected with trigonometric function definitions. **OBJECTIVES:** The student will be able to: 1. Define the six triangular trigonometric functions of an angle in a right triangle. 2. Define the six circular trigonometric functions of an angle in standard position. 3. Make the connection between the triangular and circular trigonometric functions. 4. Recognize and draw an angle in standard position. 5. Show how a point on the terminal side of an angle determines its reference triangle.	DesmosActivity: Creating Unit Circle "Trigonometry and the Astrolabe" – University of Michigan website The Astrolabe	**Activities:** • Textbook Sections 5.1-5.3 pp. 459-499 • TI website: "Linear vs. Angular Speed Lab" "Ratios of Right Triangles" Trigonometry for Solving Problems Unit Circle Project

Number of	Topics and Essential Questions	REQUIRED Critical	Additional Instructional Resources

<table>
<tr><th>Blocks</th><th></th><th>Thinking Lessons</th><th></th></tr>
<tr><td>4</td><td>AFM.8 The student will be able to, given the value of one trigonometric function, find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied. The student will find the values of the trigonometric functions of the special angles and their related angles as found in the unit circle without the aid of a calculating utility. This will include converting radians to degrees and vice versa.

<u>OBJECTIVES:</u> The student will be able to:
1. Given one trigonometric function value, find the other five trigonometric function values.
2. Use a calculator to find the value of any trigonometric function and inverse trigonometric function.
3. Develop the unit circle, using both degrees and radians.
4. Solve problems, using the circular function definitions and the properties of the unit circle.
5. Recognize the connections between the coordinates of points on a unit circle and
o coordinate geometry;
o cosine and sine values; and
o lengths of sides of special right triangles (30°-60°-90° and 45°-45°-90°).
6. Find trigonometric function values of special angles and their related angles in both degrees and radians.
7. Apply the properties of the unit circle without using a calculator.
8. Use a conversion factor to convert from radians to degrees and vice versa without using a calculator.</td><td>“Investigating the Unit Circle”- Mathematical Investigations

Desmos: Unit Circle

Desmos: Sine Function and Unit Circle</td><td>Activities:

• Textbook Section 5.4 pp. 499-511
• TI website: “When a Ruler Isn’t Enough”</td></tr>
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Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
5	**Trigonometric Functions** **AFM.9** The student will be able to, given one of the six trigonometric functions in standard form (e.g., y = A sin(Bx + C) + D where A, B, C, and D are real numbers), will • state the domain and the range of the function; • determine the amplitude, period, phase shift, and vertical shift; and • sketch the graph of the function by using transformations for at least a one-period interval. The graphing calculator will be used to investigate the effect of changing A, B, C, and D on the graph of a trigonometric function. **OBJECTIVES:** The student will be able to: 1. Determine the amplitude, period, phase shift, and vertical shift of a trigonometric function from the equation of the function and from the graph of the function. 2. Describe the effect of changing A, B, C, and D in the standard form of a trigonometric equation {e.g., y = A sin(Bx + C) + D or y = A cos(Bx + C) + D} . 3. State the domain and the range of a function written in standard form {e.g., y = A sin(Bx + C) + D or y = A cos(Bx + C) + D} . 4. Sketch the graph of a function written in standard form {e.g., y = A sin(Bx + C) + D or y = A cos(Bx + C) + D} by using transformations for at least one period or one cycle.	TI website: "Getting Triggy with It" **or** TI website: "Changes in Latitude - Modeling a Sine Function" **or** TI website: "Climate and Temperature: Sinusoidal Models"	**Activities:** • Textbook Sections 5.5-5.7 pp. 511-539 • Using Trigonometric Functions to Model Climate – National Institute of Water and Atmospheric Research Mathematical Investigations • Bicycle Wheels • TI website: "The Light Side of Trigonometry" Trigonometric Equations NUMB3RS Activity: Changing Sines Episode: "Counterfeit Reality"

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
2	**Inverse Trigonometric Functions** **AFM.10** The student will be able to identify the domain and range of the inverse trigonometric functions and recognize the graph of these functions. Restrictions on the domains of the inverse trigonometric functions will be included. **<u>OBJECTIVES:</u>** The student will be able to: 1. Find the domain and range of the inverse trigonometric functions. 2. Use the restrictions on the domains of the inverse trigonometric functions in finding the values of the inverse trigonometric functions. 3. Identify the graphs of the inverse trigonometric functions.		**Activities:** • Textbook Sections 6.5 pp. 591-604

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
8	**Trigonometric Identities** **AFM.11** The student will be able to verify basic		**Activities:**

	trigonometric identities and make substitutions using the basic identities. **OBJECTIVES:** The student will be able to: 1. Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. The basic trigonometric identities include • reciprocal identities; • Pythagorean identities; • sum and difference identities; • double-angle identities; and • half-angle identities.		• Textbook Sections 6.1-6.3 pp. 553-581

Semester 2

Number of Blocks	Topics and Essential Questions	REQUIRED Critical Thinking Lessons	Additional Instructional Resources
4	**Trigonometric Equations** **AFM.12** The student will be able to solve trigonometric equations that include both infinite solutions and restricted domain solutionsand solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, to check for reasonableness of results, and to verify algebraic solutions. **OBJECTIVES:** The student will be able to: 1. Solve trigonometric equations with restricted domains algebraically and by using a graphing utility. 2. Solve trigonometric equations with infinite solutions algebraically and by using a graphing utility. 3. Check for reasonableness of results, and verify algebraic solutions, using a graphing utility.		**Activities:** • Textbook Sections 6.6 pp. 604-620 pp. 615-616 #93-100
8	**Trigonometric Applications** **AFM.13** The student will be able to identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines. **OBJECTIVES:** The student will be able to: 1. Write a practical problem involving triangles. 2. Solve practical problems involving triangles. 3. Use the trigonometric functions, Pythagorean Theorem, Law of Sines, and Law of Cosines to solve practical problems. 4. Identify a solution technique that could be used with a given problem. 5. Find the area of a triangle and use Herron's Formula.	The Discus Throw- Pacesetter Mathematics	**Activities:** • Textbook Sections 7.1-7.2 pp. 627-649
2 blocksAssessments, Enrichment, and Remediation			

Additional information about math vocabulary can be found in the

VDOE Vocabulary Word Wall Cards

(click link above)